Determination of Sea Level Height Variation by Dynamics Crossover Adjustment

Luying Cheng, and Hou-tse Hsu
Institute of Geodesy and Geophysics Chinese Academy of Sciences, WuHan, 430077,China
Pengliu2001@x263.net

Abstract. The satellite radar altimeter surveys global mean sea level height continuously. The crossover point adjustment is based on the ascending-tracks and the descending-tracks of satellite orbit. According to the dynamic model of the altimetry satellite motion, the method of dynamic crossover point adjustment is introduced in this paper. It is the fact that the altimetry data sets in the crossover points are just as the repeated observations which constructs the basis of the crossover adjustment. Assume that h_0 is a sea level height variation, $\Delta\overline{X}_j^d$ is the satellite orbit change at time t_j^d, $\Delta\overline{X}_i^a$ is the satellite orbit change at time t_i^a, h_{ij}^{ad} is the undulation of the sea level height in the crossovers of the ascending-tracks and the descending-tracks. The sea level height variation h_0 can be gained by least-squares adjustment procedure. By establishing the dynamic model of crossover adjustment, the orbit errors could be reduced or weaken, thus improves the accuracy of the sea level height variation determined.

Keywords. Altimetry - Satellite orbit - Sea level height – Crossover adjustment

1 Introduction

Since satellite altimetry technique came out, it has widely been utilized in geodesy, geophysics and oceanographic. The sea to seize 70% of the earth area has been understood by using satellite altimetry data in large range with high accuracy. By accurately determining altimetry satellite orbit, we can obtain ocean geoid and sea surface topography based on the height from the satellite to an instantaneous sea surface (Denker et al 1990). On the bases of the altimeter accuracy raising, the earth gravity model improving and the technique of determining satellite orbit developing, more and more applications have appeared in satellite altimetry field.

According to basic satellite altimetry mathematics model, sea surface height (SSH) can be shown as (Engelis 1987)

$$h = h_s - \rho + \varepsilon \qquad (1)$$

where, h is SSH; h_s is satellite's geodetic height; ρ is the distance from satellite to the sea surface, which has been corrected by altimeter deviation, ionosphere, dry and humid troposphere, electromagnetic deviation, converse air pressure, polar tide, solid tide and ocean tide; ε is measurement noise and ocean wave noise. It is obvious, from (1), that h_s from the satellite orbit files directly affects the accuracy of SSH. In other word, the accuracy of the orbit of the altimetry satellite, especially in radial, controls the accuracy of SSH to a great extent. Since satellite altimetry technique was applied, the geodesists have been attempting to improving the satellite orbit determination conditions, raising the orbit accuracy and studding the data processing methods. The crossover adjustment is an important means in reducing the effects of the orbit errors (Schrama 1989). This method can be used not only in the same satellite crossover adjustment, but also in combining process to several altimetry satellites.

We have known that the radial orbit error of the altimetry satellite is of the long-wave characteristics, and has the feature of 1 cycle/revolution obviously (Engelis 1987; Schrama 1989; Zhai Guojun et al 1996, 1997). Therefore a polynomial or trigonometric function is usually chosen as the orbit error model, in the crossover adjustment. This kind of selection is very efficient and this process can better remove the effects of the satellite orbit error in SSH to the large number of altimetry satellites (Shum 1982; Tapley et al 1985). However, after radial orbit errors of the altimetry satellite are studied in space domain, it is found that these errors are the function of geographical position. In orbit error models stated above, pure geometry models are not able to remove the effect with geographical

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position correlation. Especially, when the earth gravitational field model is taken into account in the determination of the satellite orbit, the perturbation forms are the function of satellite's space position completely. Therefore, the orbit errors that are independent with time are dependent with the position, and influence the results of the common crossover adjustments, finally, they affect the accuracy of observations of SSH. The dynamics crossover adjustment equations are founded on the basis of the satellite orbit dynamics model and the measurement data sets at the crossover points in this paper, by which we hope to reduce the influences of the orbit errors and to improve the accuracy of SSH.

2 Dynamics Crossover Adjustment Model

The satellite altimeter can effectually measure on a large area of the ocean and provide the altimetry of the instantaneous ocean surface which in turn can be used to research the ocean variation. The satellite orbit is designed as "frozen orbit" (Born et al 1987), in order to continuously measure the global sea surface height in repeated cycle, along the same ground track. A large number of crossover points are therefore produced. These points are located almost fixed geographical position in the sea surface respectively. The observations at the crossover points can be used as repeated measurements at fixed position but different time, which is provided to us to found the dynamics crossover adjustment.

We know that, SSH is the distance from the sea surface to a reference ellipsoid. When we ignore the non-collinear properties between geocentric distance and sub-satellite point distance, SSH can be written based on (1) as

$$h = r_s - r_p - \rho + \varepsilon \tag{2}$$

where, r_s is the geocentric distance and, r_p is sub-satellite point distance. If ρ is regarded as input to equation (2), the output of the equation (SSH) can be considered as a new "observation". At this rate, SSH will be the function of the satellite position.

Based on the principle of satellite kinematics, the equation of satellite motion in inertial geocentric coordinate can be written as (Kaula 1966; Tapley et al 1985)

$$\dot{\bar{r}} = \bar{v}$$

$$\dot{\bar{r}} = -\frac{GM}{r}\frac{\bar{r}}{r} + \bar{P}(\bar{r}, \bar{v}, \bar{p}_d, t) \equiv \bar{F} \tag{3}$$

$$\bar{r}(t_0) = \bar{r}_0 \qquad \dot{\bar{r}}(t_0) = \bar{v}(t_0) = \dot{\bar{r}}_0$$

where, $\bar{r}$, $\dot{\bar{r}}$ or $\bar{v}$ and $\bar{F}$ are satellite's position vector, velocity vector and acceleration vector respectively. $\bar{P}$ is satellite's perturbation. p_d is kinematics parameter vector, including gravitational field coefficients, air drag coefficients, solar radiant pressure coefficients, etc, which are to be estimated. In precise determination of the satellite orbit, the satellite status vector can be defined as

$$\bar{X} = (\bar{r}, \bar{v}, \bar{p}_d)^T \tag{4}$$

At the same time, the status equation is written as

$$\dot{\bar{X}} = G(\bar{X}, t) \qquad \bar{X}(t) = \bar{X}_0 \tag{5}$$

the corresponding status function is

$$G(\bar{X}, t) = (\bar{v}^T, \bar{F}^T, 0)^T \tag{6}$$

Consequently, theoretical function of SSH reads

$$h_c = H(\bar{X}, t) \tag{7}$$

Assume that h_i^a is SSH observation in an ascent arc at t_i^a, h_j^d is SSH observation in a descent arc at t_j^d, and crossover point is thus composed. Considering the variation of SSH, we assume that SSH varies in linear in one repeated cycle (excluding ERS-1/168 data), then we have

$$h_j^d - h_i^a = h_0\left(t_j^d - t_i^a\right) + H\left(\bar{X}_j^d, t_j^d\right) - H\left(\bar{X}_i^a, t_i^a\right) \tag{8}$$

Linearizing (8), we obtain error equations

$$v_{ij}^{ad} = h_0\left(t_j^d - t_i^a\right) + \frac{\partial H}{\partial \overline{X}_j^d}\Delta\overline{X}_j^d - \frac{\partial H}{\partial \overline{X}_i^a}\Delta\overline{X}_i^a + h_{ij}^{ad} \qquad (9)$$

where, h_0 is the linear variation term of SSH in one repeated cycle, $\Delta\overline{X}_j^d$ is the satellite orbit change at time t_j^d, $\Delta\overline{X}_i^a$ is the satellite orbit change at time t_i^a, h_{ij}^{ad} is the undulation of SSH at the crossover point of ascending-tracks and descending-tracks.

We know by satellite motion theory that, the satellite's position vector and velocity vector can be represented by the function of position, velocity and force parameters at satellite initial epoch. Therefore, after the linearization of the satellite status equation, we have

$$\Delta\dot{\overline{X}} = \frac{\partial G}{\partial \overline{X}}\Delta\overline{X} = D\Delta\overline{X} \qquad (10)$$

Now, we introduce a status transfer matrix

$$\Phi(t, t_0) = \frac{\partial \overline{X}}{\partial \overline{X}_0} \qquad (11)$$

so that

$$\Delta\overline{X} = \Phi(t, t_0)\Delta\overline{X}_0 \qquad (12)$$

where, $\Delta\overline{X}_0$ is initial satellite status and $\dot{\Phi}$ follows

$$\dot{\Phi}(t, t_0) = D\Phi(t, t_0) \qquad (13)$$

let

$$\frac{\partial H}{\partial \overline{X}_j^d} = N_j^d, \qquad \frac{\partial H}{\partial \overline{X}_i^a} = N_i^a \qquad (14)$$

and

$$\overline{N}_j^d = N_j^d\Phi\left(t_j^d, t_0\right)$$

$$\overline{N}_i^a = N_i^a\Phi\left(t_i^a, t_0\right) \qquad (15)$$

$$\overline{N}_{ij}^{ad} = \overline{N}_j^d - \overline{N}_i^a$$

The observation equation can be written as

$$v_{ij}^{ad} = h_0\left(t_j^d - t_i^a\right) + \overline{N}_{ij}^{ad}\Delta\overline{X}_0 + h_{ij}^{ad} \qquad (16)$$

In one repeated cycle, m ascending-tracks and n descending-tracks can consist of $m \times n$ crossover points, which can be treated as $m \times n$ observations at $m \times n$ stations in global ocean. The matrix form of observation equations is

$$V = \begin{pmatrix} T & N \end{pmatrix}\begin{pmatrix} h_0 \\ \Delta\overline{X}_0 \end{pmatrix} + \overline{h} \qquad (17)$$

where

$$T = \left(t_1^d - t_1^a, t_1^d - t_2^a, \cdots, t_1^d - t_m^a, t_2^d - t_1^a, t_2^d - t_2^a, \cdots, t_n^d - t_m^a\right)^T$$

$$N = \left(\overline{N}_{11}^{ad}, \overline{N}_{21}^{ad}, \cdots, \overline{N}_{m1}^{ad}, \overline{N}_{12}^{ad}, \overline{N}_{22}^{ad}, \cdots, \overline{N}_{mn}^{ad}\right)^T$$

By least-square adjustment, the SSH variation h_0 in one repeated cycle is easily obtained.

3 Discussion and Conclusion

By using satellite altimeter, we can continuously measure the height of the global ocean. In one repeated cycle, a large number of crossover points are formed in sea surface by ascending-tracks and descending-tracks. Their positions are almost fixed regularly. The observations at crossover points can be treated as repeated measurements in the same positions at different time, which constructs the foundation of the dynamics crossover adjustment for the SSH variations.

The SSH observations are derived by the distance from the satellite to the sea surface. The noises of the altimeter, sea wave and the satellite orbit errors are included in SSH. Based on the reference [10], the radial errors of altimetry satellite orbit in space domain can be divided to two parts. One part is related to the ascending-tracks and descending-tracks, and the magnitudes for both tracks should be equivalent, but the signs are reverse. Another part is not relevant to ascending-tracks or descending-tracks, and the magnitudes are also equivalent, and the signs are the same. The two parts are related with the geographical position of the satellite, but not related with the satellite's initial position. These provide us some beneficial computation condition. Firstly, when the crossover points are formed by ascending-tracks and descending-tracks, the orbit error parts could be removed by orbit difference in error equations in the dynamical crossover adjustment, the other errors can be weakened in the adjustment with redundant observations. Therefore, the two parts of orbit errors do not affect the computation results in the dynamical crossover adjustment. Secondly, because the two parts of errors are only related with the satellite's position, in error equations, the variation of SSH is mainly produced at time difference $t_j - t_i$, which is very beneficial for us to solve parameters of SSH variation.

On the basis of the satellite motion theory, satellite position and velocity at any time can be expressed as the function of satellite initial position, velocity and dynamical parameters. In SSH "observations" derived, it is essential that accurate satellite positions or improved initial epoch status should be known. In the dynamical crossover adjustment, the error equations are formed by the distance difference from the satellite to the sea surface at crossover points, the ambiguity thus will be produced at determining the initial orbit status. It indicates that the dynamical crossover adjustment can't be used in improving the satellite orbit. Further more, we can see that crossover point undulations include a part of orbit perturbation variation. It is an advantage that the orbit perturbation can be reduced by the dynamical crossover adjustment in the determination of SSH variation.

The dynamical crossover adjustment is different from the integrated adjustment. At first, the purpose is different, thus the problems to be solved are also different. The dynamical crossovers adjustment method completely utilizes satellite altimeter data, especially, uses the reliable parameters at crossover points as controlling parameters. It should be pointed that the dynamical crossover adjustment method can only be applied to solve high accurate relative values. If precise geodetic or oceanographic parameters as SSH are needed, it is essential to input new external observations.

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